

Module 2 DC Circuit

Lesson 5

Node-voltage analysis of
resistive circuit in the
context of dc voltages
and currents

Objectives

- To provide a powerful but simple circuit analysis tool based on Kirchhoff's current law (KCL) only.

L.5.1 Node voltage analysis

In the previous lesson-4, it has been discussed in detail the analysis of a dc network by writing a set of simultaneous algebraic equations (based on KVL only) in which the variables are currents, known as mesh analysis or loop analysis. On the other hand, the node voltage analysis (Nodal analysis) is another form of circuit or network analysis technique, which will solve almost any linear circuit. In a way, this method completely analogous to mesh analysis method, writes KCL equations instead of KVL equations, and solves them simultaneously.

L.5.2 Solution of Electric Circuit Based on Node Voltage Method

In the node voltage method, we identify all the nodes on the circuit. Choosing one of them as the reference voltage (i.e., zero potential) and subsequently assign other node voltages (unknown) with respect to a reference voltage (usually ground voltage taken as

zero (0) potential and denoted by (---). If the circuit has “n” nodes there are “n-1” node voltages are unknown (since we are always free to assign one node to zero or ground potential). At each of these “n-1” nodes, we can apply KCL equation. The unknown node voltages become the independent variables of the problem and the solution of node voltages can be obtained by solving a set of simultaneous equations.

Let us consider a simple dc network as shown in Figure 5.1 to find the currents through different branches using “**Node voltage**” method.

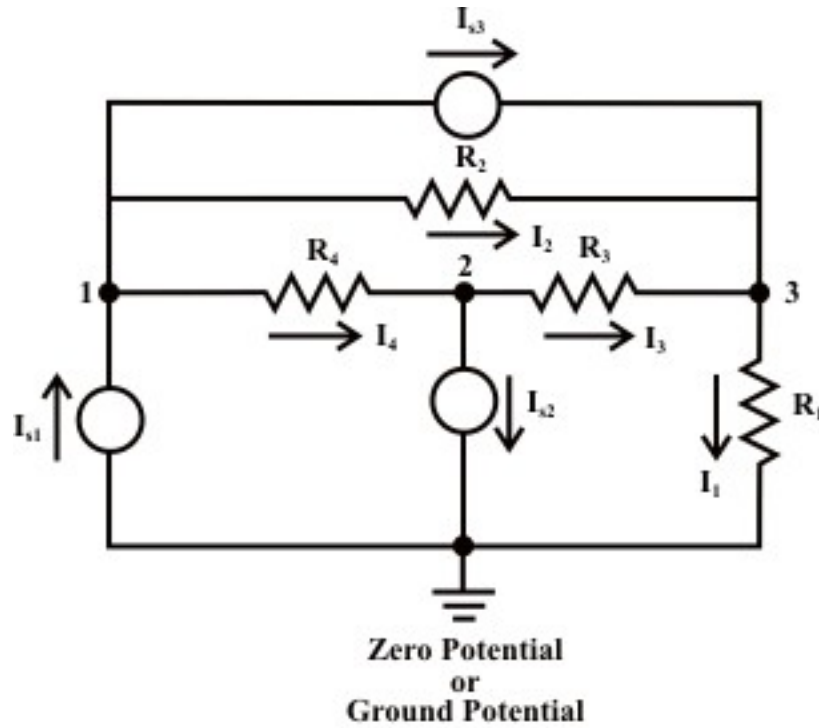


Fig. 5.1

KCL equation at “**Node-1**”:

$$I_{s1} - I_{s3} - \left(\frac{V_1 - V_2}{R_4} \right) - \left(\frac{V_1 - V_3}{R_2} \right) = 0 ; \rightarrow I_{s1} - I_{s3} - \left(\frac{1}{R_2} + \frac{1}{R_4} \right) V_1 - \left(\frac{1}{R_4} \right) V_2 - \left(\frac{1}{R_2} \right) V_3 = 0$$

$$I_{s1} - I_{s3} = G_{11} V_1 - G_{12} V_2 - G_{13} V_3 \quad (5.1)$$

where G_{ii} = sum of total conductance (self conductance) connected to Node-1.

KCL equation at “**Node-2**”:

$$\left(\frac{V_1 - V_2}{R_4} \right) - \left(\frac{V_2 - V_3}{R_3} \right) - I_{s2} = 0 ; \rightarrow -I_{s2} = -\left(\frac{1}{R_4} \right) V_1 + \left(\frac{1}{R_3} + \frac{1}{R_4} \right) V_2 - \left(\frac{1}{R_3} \right) V_3$$

$$-I_{s2} = -G_{21} V_1 + G_{22} V_2 - G_{23} V_3 \quad (5.2)$$

KCL equation at “**Node-3**”:

$$I_{s3} + \left(\frac{V_2 - V_3}{R_3} \right) + \left(\frac{V_1 - V_3}{R_2} \right) - \left(\frac{V_3}{R_1} \right) = 0 ; \rightarrow I_{s3} = -\left(\frac{1}{R_2} \right) V_1 - \left(\frac{1}{R_3} \right) V_2 + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) V_3$$

$$I_{s3} = -G_{31} V_1 - G_{32} V_2 + G_{33} V_3 \quad (5.3)$$

In general, for the i^{th} Node the KCL equation can be written as

$$\sum I_{ii} = -G_{i1} V_1 - G_{i2} V_2 - \dots - G_{iN} V_N$$

where,

$\sum I_{ii}$ = algebraic sum of all the current sources connected to 'Node- i ', $i = 1, 2, \dots, N$. (Currents entering the node from current source is assigned as +ve sign and the current leaving the node from the current source is assigned as -ve sign).

G_{ii} = the sum of the values of conductance (reciprocal of resistance) connected to the node ' i '.

G_{ij} = the sum of the values of conductance connected between the nodes ' i ' and ' j '.

Summarize the steps to analyze a circuit by node voltage method are as follows:

Step-1: Identify all nodes in the circuit. Select one node as the reference node (assign as ground potential or zero potential) and label the remaining nodes as unknown node voltages with respect to the reference node.

Step-2: Assign branch currents in each branch. (The choice of direction is arbitrary).

Step-3: Express the branch currents in terms of node assigned voltages.

Step-4: Write the standard form of node equations by inspecting the circuit. (No of node equations = No of nodes (N) - 1).

Step-5: Solve a set of simultaneous algebraic equation for node voltages and ultimately the branch currents.

Remarks:

- Sometimes it is convenient to select the reference node at the bottom of a circuit or the node that has the largest number of branches connected to it.
- One usually makes a choice between a mesh and a node equations based on the least number of required equations.

Example-L-5.1: Find the value of the current I flowing through the battery using 'Node voltage' method.

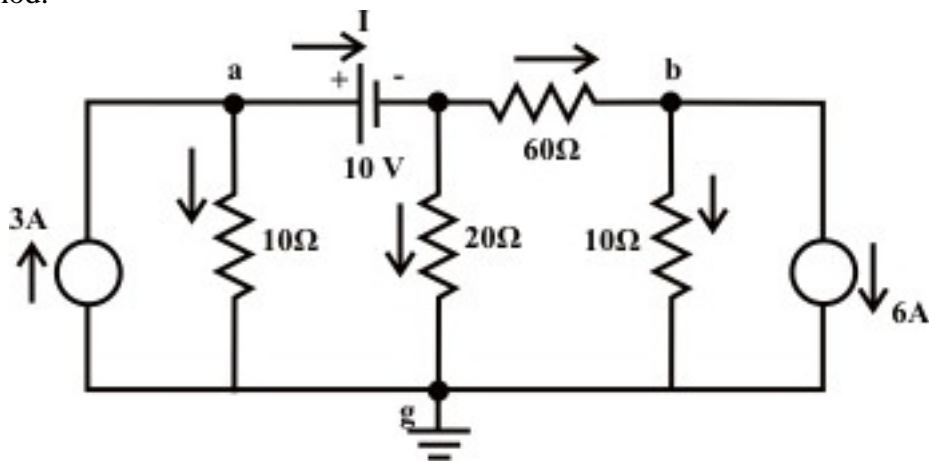


Fig. 5.2

Solution: All nodes are indicated in fig.5.2 and 'Node-g' is selected as reference voltage. If a voltage source is connected directly between the two nodes, the current flowing through the voltage source cannot be determined directly since the source voltage V_S is independent of current. Further to note that the source voltage V_S fixes the voltage between the nodes only. For the present example, the voltage of the central node is known since it is equal to $(V_a - 10) \text{ volt}$.

KCL equation at node-a:

$$3 = \frac{V_a - 0}{10} + I \rightarrow 10I + V_a = 30 \quad (5.4)$$

KCL equation at node-b:

$$\frac{(V_a - 10) - V_b}{60} = 6 + \frac{V_b - 0}{10} \rightarrow V_a - 7V_b = 370 \quad (5.5)$$

To solve the equations (5.4)-(5.5), we need one more equation which can be obtained by applying KCL at the central node (note central node voltage is $(V_a - 10)$).

$$I = \frac{V_a - 10}{20} + \frac{(V_a - 10) - V_b}{60} \rightarrow 60I = 4V_a - V_b - 40 \rightarrow I = \frac{(4V_a - V_b - 40)}{60} \quad (5.6)$$

Substituting the current expression (5.6) in equation (5.4) we get,

$$\frac{(4V_a - V_b - 40)}{6} + V_a = 30 \rightarrow 10V_a - V_b = 220 \quad (5.7)$$

Equations (5.5) and (5.7) can be solved to find $V_b = -50.43V$ and $V_a = 16.99V$.

We can now refer to original circuit (fig.5.2) to find directly the voltage across every element and the current through every element. The value of current flowing through the voltage source can be computed using the equation (5.6) and it is given by $I = 1.307 A$. Note that the current I (+ve) is entering through the positive terminal of the voltage source and this indicates that the voltage source is absorbing the power, in other words this situation is observed when charging a battery or source.

Example-L-5.2: Find the **current** through 'ab-branch' (I_{ab}) and **voltage** (V_{cg}) across the current source using Node-voltage method.

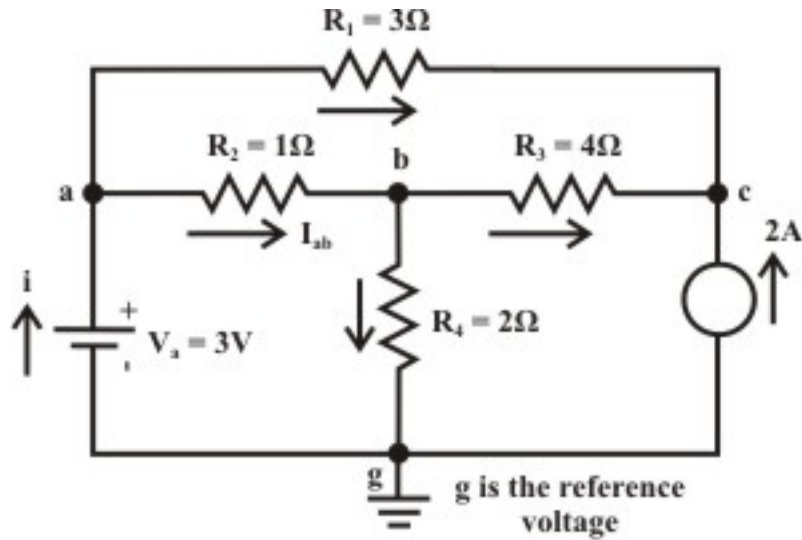


Fig. 5.3

Solution:

KCL at node-a: (note $V_a = 3V$)

$$i = \frac{V_a - V_b}{R_2} + \frac{V_a - V_c}{R_1} \rightarrow i = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) V_a - \frac{1}{R_2} V_b - \frac{1}{R_1} V_c \rightarrow i = 1.33V_a - V_b - \frac{1}{3}V_c \quad (5.8)$$

KCL at node-b: (note $V_g = 0V$)

$$\frac{V_a - V_b}{R_2} = \frac{V_b - V_c}{R_3} + \frac{V_b - V_g}{R_4} \rightarrow \left(1 + \frac{1}{4} + \frac{1}{2} \right) V_b - V_a - \frac{1}{4}V_c = 0 \quad (5.9)$$

KCL at node-c:

$$2 + \frac{V_b - V_c}{R_3} + \frac{V_a - V_c}{R_1} = 0 \rightarrow \left(\frac{1}{4} + \frac{1}{3} \right) V_c - \frac{1}{3}V_a - \frac{1}{4}V_b = 2 \quad (5.10)$$

Using the value of $V_a = 3V$ in equations (5.8)-(5.10) we get the following equations:

$$V_b + \frac{1}{3}V_c = 3.99 - i \quad (5.11)$$

$$1.75V_b - \frac{1}{4}V_c = 3 \quad (5.12)$$

$$0.583V_c - \frac{1}{4}V_b = 3 \quad (5.13)$$

Simultaneous solution of the above three equations, one can get $V_c = 6.26V$, $V_b = 2.61V$

and hence $I_{ab} = \frac{V_a - V_b}{R_2} = \frac{3 - 2.61}{1} = 0.39A$ (current flowing in the direction from 'a' to 'b').

Example-L-5.3 Determine the current, i shown in fig. 5.4 using node-voltage method ---
(a) applying voltage to current source conversion (b) without any source conversion.

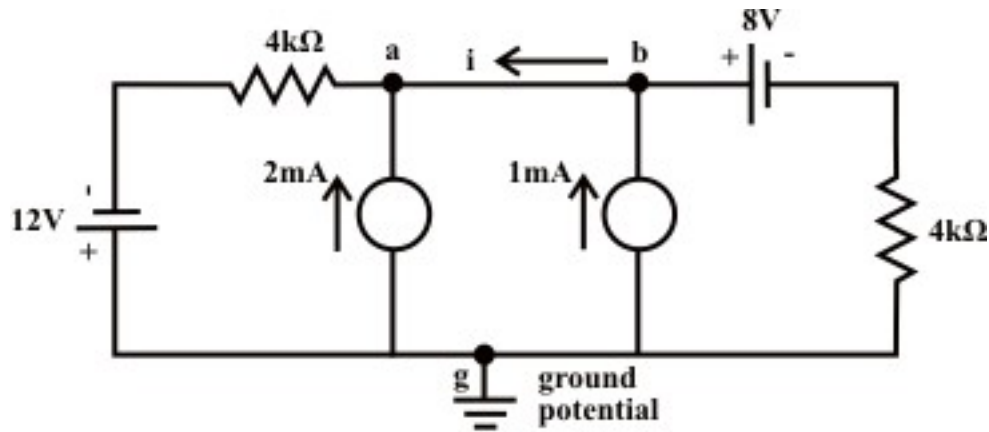


Fig. 5.4

Solution:

Part(a):

In node voltage analysis, sometimes the solution turns out to be very simple while we change all series branches containing voltage sources to their equivalent current sources. On the other hand, we observed in the loop analysis method that the conversion of current source to an equivalent voltage makes the circuit analysis very easy (see example-L4.2) and simple. For this example, both the practical voltage sources (one is left of 'node-a' and other is right of 'node-b') are converted into practical current sources. After transformation, the circuit is redrawn and shown in fig. 5.5(a).

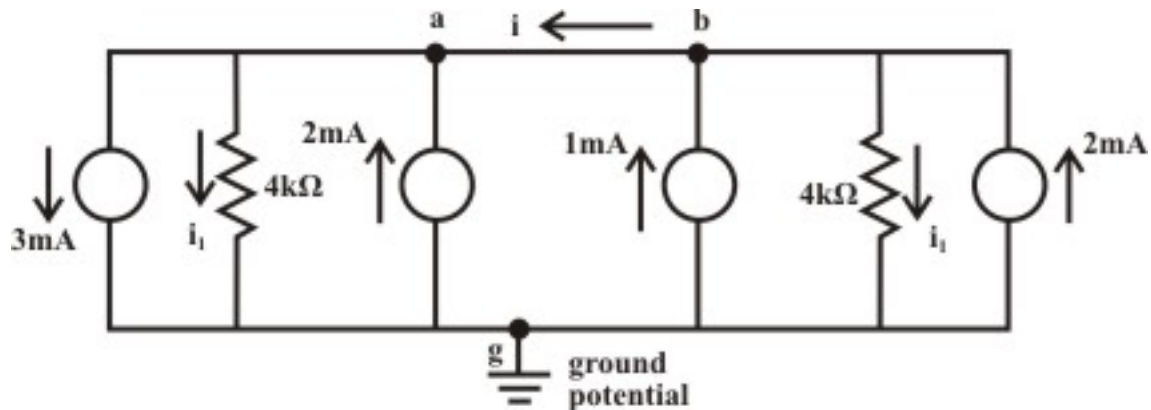


Fig. 5.5(a)

KCL at node 'b':

$$i + i_1 = 2 + 1 = 3 \quad (5.14)$$

KCL at node 'a':

$$i + 2 = 3 + i_1 \rightarrow i - i_1 = 1 \quad (5.15)$$

From equations (5.14)-(5.15), one can get $i = 2\text{mA}$ (current flows from 'b' to 'a') and $i_1 = 1\text{mA}$.

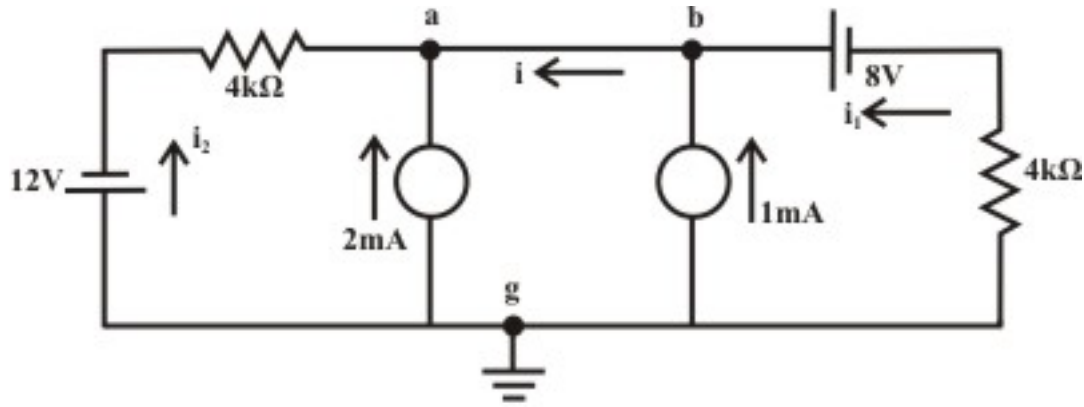


Fig. 5.5(b)

Part(b):

Let us assume i_1 is the current flowing through the 8V battery source from 'right to left' and i_2 is the current flowing through the 12V battery source from 'bottom to top' (see Fig. 5.5(b)).

KCL at node 'b': It is assumed that the current flowing in $4k\Omega$ resistor from bottom to top terminal. This implies that the bottom terminal of $4k\Omega$ resistor is higher potential than the top terminal. (currents are in mA, note $V_a = V_b$)

$$i = 1 + i_1 \rightarrow i = 1 + \frac{0 - (V_a - 8)}{4} \quad (5.16)$$

KCL at node 'a': (currents are in mA)

$$i + i_2 + 2 = 0 \rightarrow i = -i_2 - 2 \rightarrow i = -\left(\frac{-12 - V_a}{4}\right) - 2 \quad (5.17)$$

From (5.16) and (5.17), we get $V_a = 4V$ and $i = 2mA$ (current flows from 'b' to 'a').

L.5.3 Test Your Understanding

[Marks: 50]

T.5.1 Node analysis makes use of Kirchhoff's----- law just as loop analysis makes use of Kirchhoff's ----- law. [1]

T.5.2 Describe a means of telling how many node voltage equations will be required for a given circuit. [1]

T.5.3 In nodal analysis how are voltage sources handled when (i) a voltage source in a circuit is connected between a non-reference node and the reference node (ii) a voltage source connected between two non-reference nodes in nodal analysis. [4]

T.5.4 A voltage in series with a resistance can be represented by an equivalent circuit that consists of ----- in parallel with that ----- . [2]

T.5.5 The algebraic sum of the currents ----- in a node must be equal to the algebraic sum of currents ----- the node. [2]

T.5.6 Apply node voltage analysis to find i_0 and the power dissipated in each resistor in the circuit of Fig.5.6. [10]

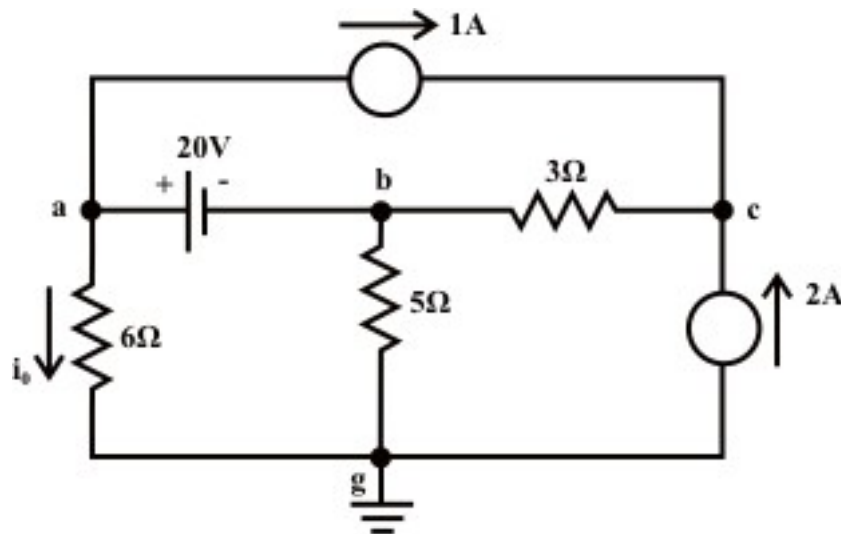


Fig. 5.6

(Ans. $i_0 = 2.73\text{ A}$, $P_6 = 44.63\text{ W}$, $P_5 = 3.8\text{ W}$, $P_3 = 0.333\text{ W}$ (note $\rightarrow V_c = 5.36\text{ V}$, $V_b = 4.36\text{ V}$))

T.5.7 For the circuit shown in fig. 5.7, find V_a using the node voltage method. Calculate power delivered or absorbed by the sources. [10]

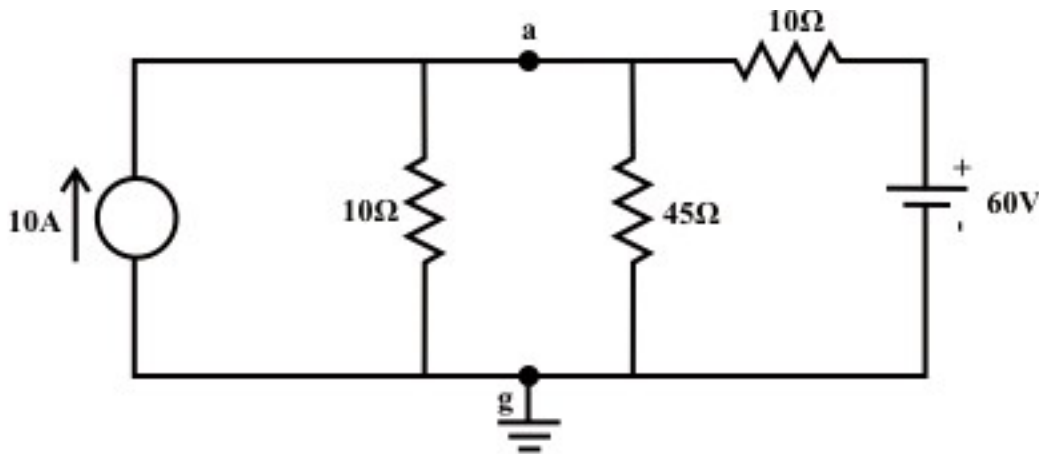


Fig. 5.7

(Answer: $V_a = 72\text{ V}$, $P_{(\text{voltage source})} = 72\text{ W (absorbed)}$, $P_{(\text{current source})} = 201.8\text{ W (delivered)}$)

T.5.8 Using nodal analysis, solve the voltage (V_x) across the 6A current source for the circuit of fig. 5.8. Calculate power delivered or absorbed by the sources [10]

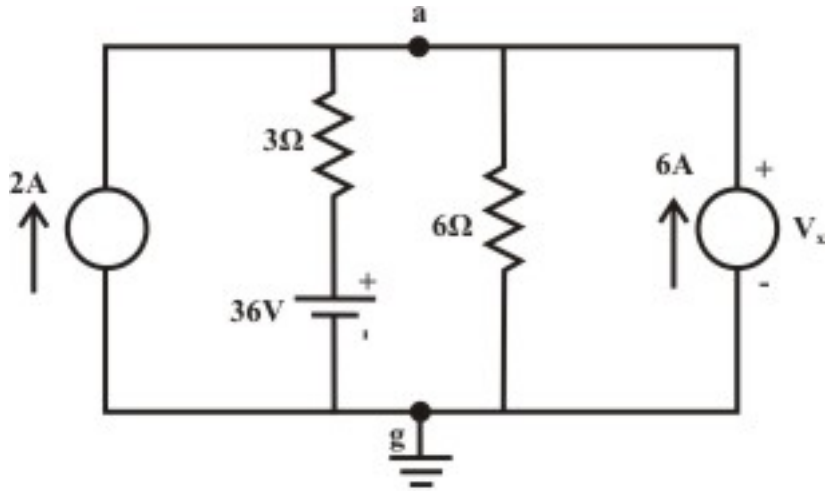


Fig. 5.8

(Answer: $V_a = 60V$, $P_{(12A \text{ ideal current source})} = 720W(\text{delivered})$,
 $P_{(\text{ideal voltage source})} = 288W(\text{absorbed})$, $P_{(6A \text{ ideal current source})} = 360W(\text{delivered})$)

T.8 Determine the voltage across the 10Ω resistor of fig. 5.9 using nodal analysis. [10]

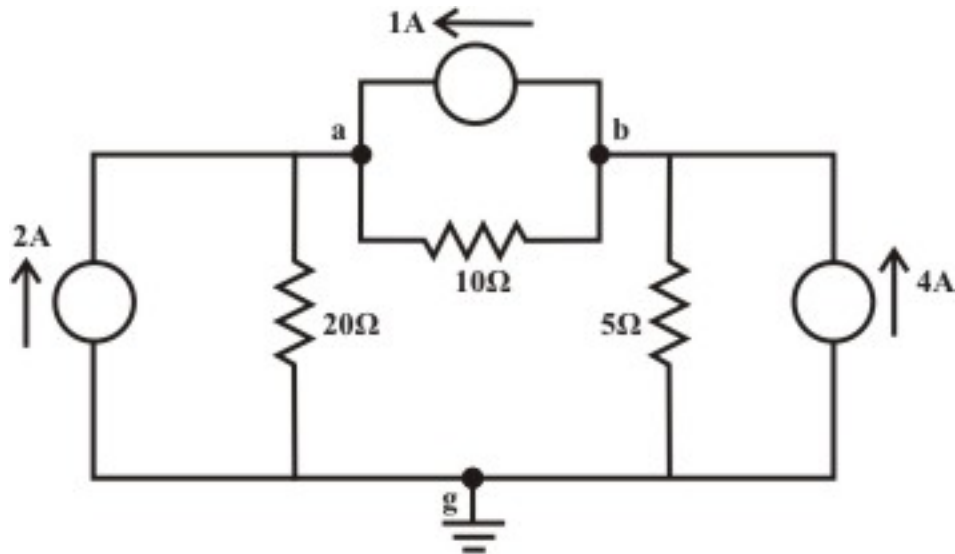


Fig. 5.9

(Answer: $V_{ab} = 34.29V$ (a is higher potential than b))